**Things to present**

**Physical consideration:**

1. The object of this paper is to propose an energy based fatigue approach which handles multidimensional time varying loading histories.
2. We assume that the energy dissipated at small scales governs fatigue at failure. The basis of our model is to consider a plastic behavior at the mesoscopic scale. The yield function depends on the deviatoric and the hydrostatic part. A kinematic hardening under the assumption of associative plasticity is also considered.
3. We also follow the Dang Van paradigm at macro scale. The structure is elastic at the macroscopic scale. At each material points, there is a stochastic distribution of weak points which will undergo strong plastic yielding, which contribute to energy dissipation without affecting the overall macroscopic stress.
4. Instead of using the number of cycles, we use the concept of loading history. Fatigue will then be determined from the plastic shakedown cycle.

**Statistics method:**

1. In more details, at each scale s of a plastic evolution process there is a weakened yield limit, zero initial plastic strain and zero initial backstress at initial time.
2. The choice of a power law has two reasons: on one hand, this type of distribution corresponds to a scale invariant process, on the other hand it leads in cyclic loading to a prediction of a number of cycles to life limit as a power law function of the stress intensity.

**Mean stress:**

1. Positive mean stress clearly reduces the fatigue life of the material. In design evaluation of multiaxial fatigue with mean stress, a simplified, conservative relation between mean stress and equivalent alternating stress is necessary. We can improve the model by modifying the yield function and the localization tensor.
2. with $\uline{\uline{S}}(s)$ denoting the deviatoric part of the stress tensor at microscale, and $\uline{\uline{b}}(s)$ the corresponding backstress at the same scale.

**Local plastic model:**

1. First we should describe the mesoscopic stress state. The model considers a plastic behavior at the mesoscopic scale. The mesoscopic stress evolution equations are thus:

**Wcyc:**

1. We can then calculate the local dissipated energy W at point M during one cycle by cumulating the input of all sub-scales which result in plastic regime with their probabilities
2. If the dissipated energy accumulates linearly until a failure value WF, we can get directly the time to failure
3. We should introduce below a method to handle such a nonlinearity.

**Nonlinearity:**

1. This is a nonlinear law but used with a constant $\alpha$, there will be no sequence effect. In other words, when applying two successive cycles of different intensities, the failure will occur at the same number of cycles whatever the order of the loading (high then low versus low then high).

**Limitations:**

1. Our first approach takes one cycle as unit time. We compute analytically the energy dissipation at each scale during this cycle. The method is valid for simple loading history and which includes the integration on all weakening scales. The damage $D$ is accumulated after each cycle.
2. However, there are certain limitations of this method. Firstly we need a load history decomposition in cycles. Secondly in real life the perfect close loop cycle is hardly applicable. The idea is then to replace the relative increment of dissipated energy per cycle by the relative increment of dissipated energy per unit time:

**Gaussian:**

1. Thus we propose a more general method which can be integrated by a step by step strategy. We compute numerically the dissipation at different scales using an implicit Euler time integration of the constitutive laws of section 1.4. After which we make a numerical integration on different scales. Then we can update the damage and go to next time step.
2. Instead of doing the scale integration directly which can be difficult for complex loading, the Gaussian Quadrature rule with Legendre points is used to give the value of local dissipated energy rate. The use of Gaussian Quadrature rule changes the integrand s from infinity to finite fixed values without affecting the integration results.

**Regime determination:**

1. At time t, the material could be both in elastic and plastic regime at different scales.

**Multi-dimensional application:**

1. In real case, the vertical force $F\_z$ is much larger than the axial and horizontal forces $F\_x$ and $F\_y$. We first scale the axial and horizontal forces to reach comparable impact and transform them in principal stresses Fx applied along the stress principle vector $ \alpha$(respectively $\beta$) that we choose randomly.